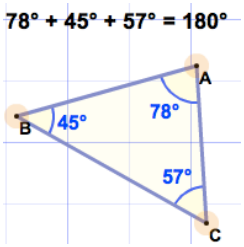
Introduction to Circle Theorem

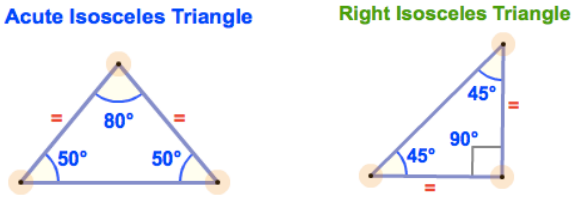
It is a well-known fact that **circle theorem** is not the most liked topic on the cxc exam and I recall that fact strongly as I remember learning it in secondary school, however if well understood, I believe it has the potential to be a very entertaining topic. This section of geometry relies strongly on the **memory** (yes more memory than calculation) of a few key geometric principles.

Here are a few tips to solve the majority of these problems. Always observe the diagrams carefully and be on the lookout for the following:

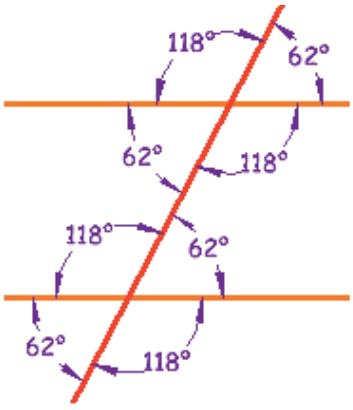
1. The sum of the three angles in a triangle is always equal to 180^0.



2. For the case of the Isosceles triangle, its base angles are equal.



3. When parallel lines get crossed by another line (which is called a Transversal), you can see that many angles are the same, as in this example:



4. The sum of the interior angles of a quadrilateral (Four-sided figure) is always equal to 360 degrees.

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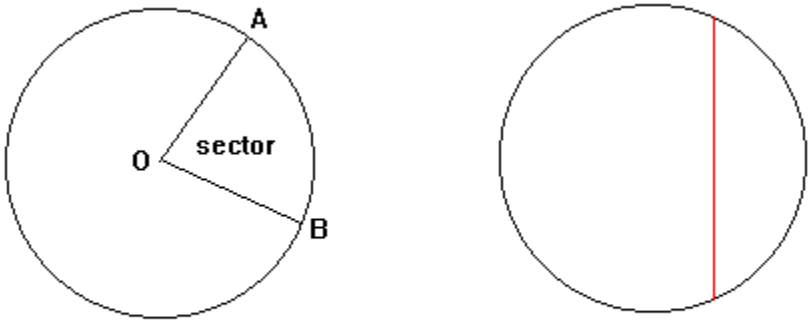
Circles

A circle is a set of points, which are all a certain distance from a fixed point known as the center.

A line joining the center of a circle to any of the points on the circle is known as a *radius*.

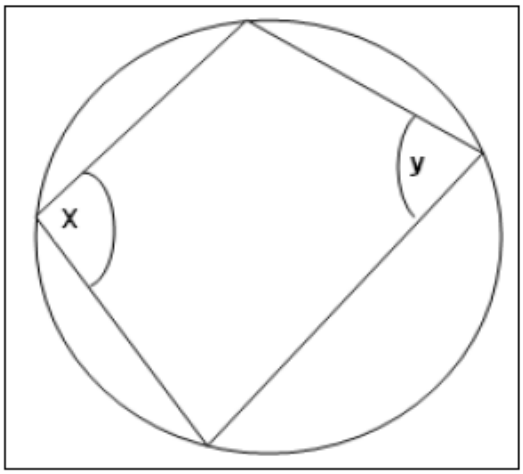


In circle no.1 **AB** is part of the circumference and is known as an **arc** and the area in between them is called a **sector**. In circle no.2 the red line in the second diagram is called a **chord**. It divides the circle into a **major segment** and a **minor segment**.

Circle Theorems

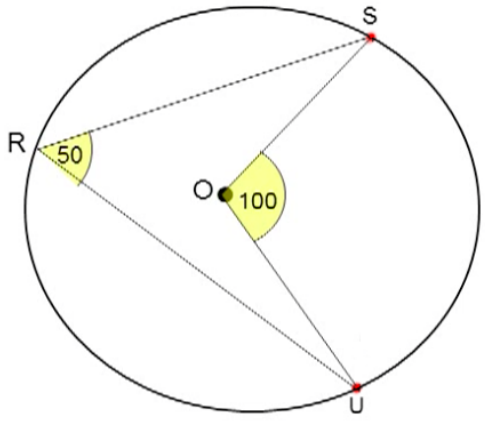
Now that you have reviewed the basics of geometry we will now deal with the rules that govern circle theorem.

5. **Cyclic quadrilaterals** (i.e. four-sided figures formed within an enclosed circle)
   The Rule States: Opposite Angles are Supplementary



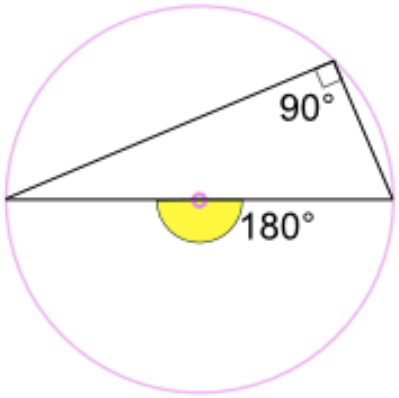
$\sphericalangle x + \sphericalangle y = 180$

6. **Angles formed at the center of the circle are twice the size of the angle formed at the circumference.**

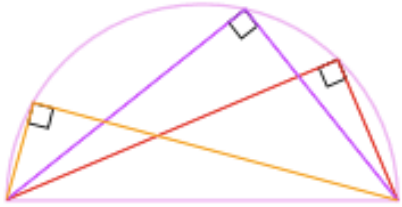


NB. This rules holds when the two angles are formed from two points on the circumference of the circle (i.e the points labeled S and U in red on the diagram).

7. **The angles formed within a semicircle are always equal to 90^0.**

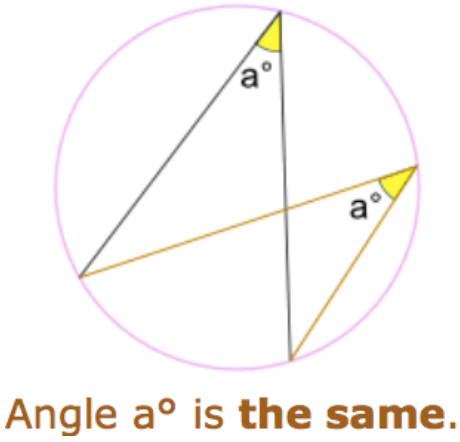


NB. The end points are at either ends of the circles diameter and the apex point can be anywhere on the circumference. **See Below**



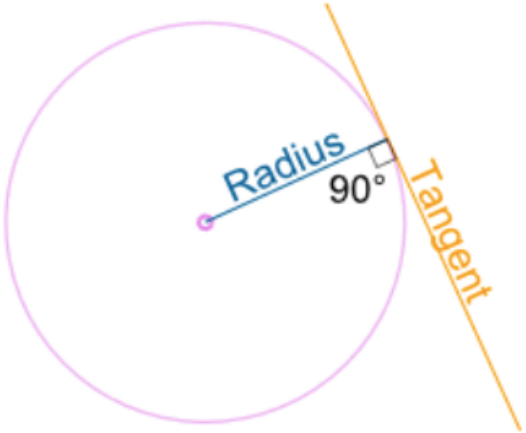
So there we go! No matter **where** that angle is on the circumference, it is **always 90°**

8. **Angles in the same segment are equal** or it is also stated as **angles subtended on the same arc are equal.** (Usually very tricky to see).



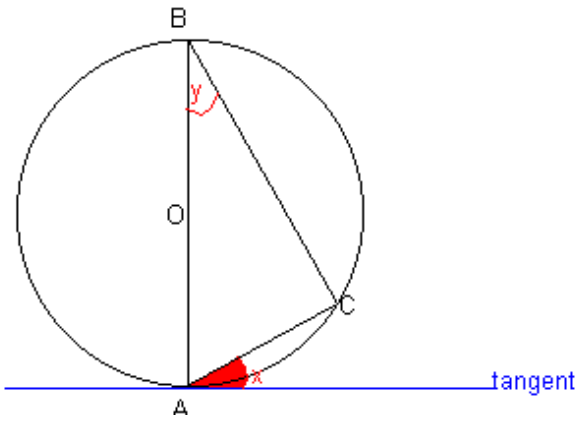
9. A **tangent line** is a straight line, which touches the circle at **only one point** (so it does not cross the circle- it just touches it). It always forms a right angle with the circle's radius, at the point of contact.

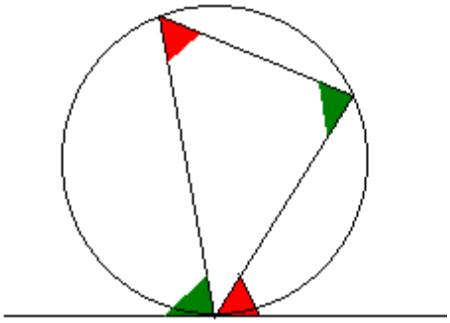
The Rule States: The Angle between radius and tangent = 90^0.



This last rule (No. 10) is the trickiest of them all but with practice it becomes easier to spot. It again deals with tangents but this time we will not use the radius line.

10. The angle between tangent and chord is equal to angle in the alternate segment.





SUMMARY:

1. Sum of all angles (Triangle = 180^0)
2. Isosceles triangle (base angles are equal)
3. Parallel lines - alternate angles equal, (Z angles)
4. Cyclic quadrilateral (opposite Angles are Supplementary)
5. Angles at the center = twice angle at circumference.
6. Angles in a semicircle = 90^0
7. Angles in the same segment are equal
8. Angle between radius and tangent = 90^0
9. Angle between tangent and chord = angle in alternate segment

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REFERENCES:

Mathisfun.com

cxcDirect.org (Circle Theorem)